EXERCISE 2 : ELECTRON SPIN RESONANCE

$$\mathcal{H} = g\mu_B \vec{B}\cdot\vec{S}$$

1) $\vec{B} = (0, 0, B_z)$

$$\to \hat{H} = g\mu_B B_z \hat{S}_z = \frac{1}{2} g\mu_B B_z \hat{\sigma}_z = \frac{1}{2} g\mu_B \begin{pmatrix} B_z & 0 \\ 0 & -B_z \end{pmatrix}$$

$$E_\pm = \pm \frac{1}{2} g\mu_B B_z$$

$+\frac{1}{2} g\mu_B B_z$

0

$-\frac{1}{2} g\mu_B B_z$

$$\boxed{\Delta E = E_{Zeeman} = g\mu_B B_z}$$

$E_Z > k_B T$ AND $E_Z < E_c$

$$g\mu_B B_z > k_B T \to B_z > \frac{k_B T}{g\mu_B}$$

@ 10 mK : $B_z > \frac{0{,}86\ \mu eV}{0{,}44 \cdot 58\ \mu eV} T = 34\ mT$ (for GaAs)

2) $$H = \frac{1}{2} g\mu_B \left(B_z \sigma_z + B_x \cos(\omega t + \varphi)\sigma_x \right) =$$

$$= \boxed{\frac{1}{2} g\mu_B \begin{pmatrix} B_z & B_x \cos(\omega t + \varphi) \\ B_x \cos(\omega t + \varphi) & -B_z \end{pmatrix}}$$

$$\Delta E_z = h\nu \to \nu = \frac{\Delta E_z}{h} = \frac{g\mu_B B_z}{h} = \frac{0{,}44 \cdot 58\ \mu eV \cdot 1}{4{,}1 \cdot 10^{-15}\ eV} = 6.2\ GHz$$

3) $H = \frac{1}{2} g\mu_B \begin{pmatrix} B_z & B_x\cos(\omega t+\varphi) \\ B_x\cos(\omega t+\varphi) & -B_z \end{pmatrix}$

In the rotating frame: $H = -\frac{\hbar}{2}\begin{pmatrix} -\Delta & \Omega e^{-i\varphi} \\ \Omega e^{i\varphi} & \Delta \end{pmatrix}$

If we follow exactly the same steps of the first exercise sheet, it is possible to show that:

$\Delta = \omega - \omega_0 = \omega - \frac{g\mu_B B_z}{\hbar}$ → detuning

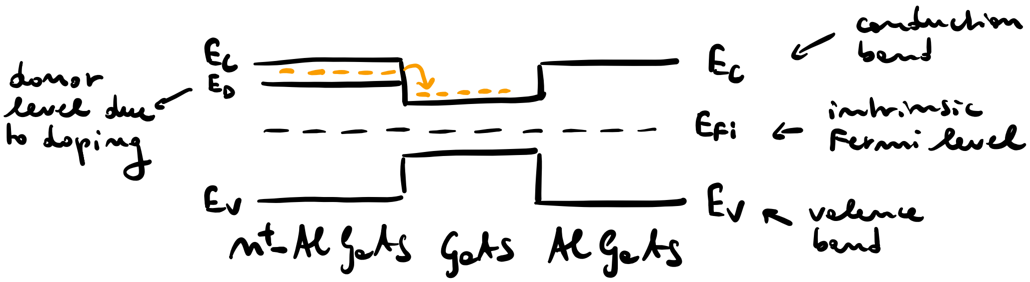
$\Omega = \frac{1}{2}\frac{g\mu_B B_x}{\hbar}$ → Rabi frequency

π pulse: $\Omega t = \pi \rightarrow t = \frac{\pi}{\Omega} = \frac{2\pi\hbar}{g\mu_B B_x}$

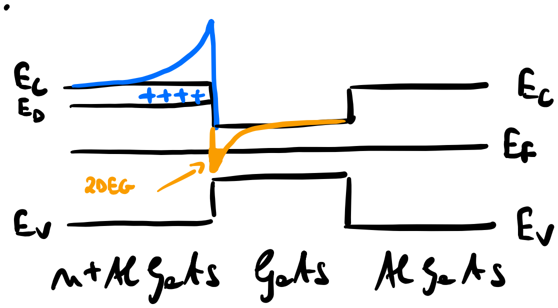
$\rightarrow B_x = \frac{h}{g\mu_B t} = 40{,}6 \text{ mT}$

EXERCISE 3: IDENTIFYING COMPONENTS ON A REAL QUANTUM DOT DEVICE

1) z- confinement → heterostructure



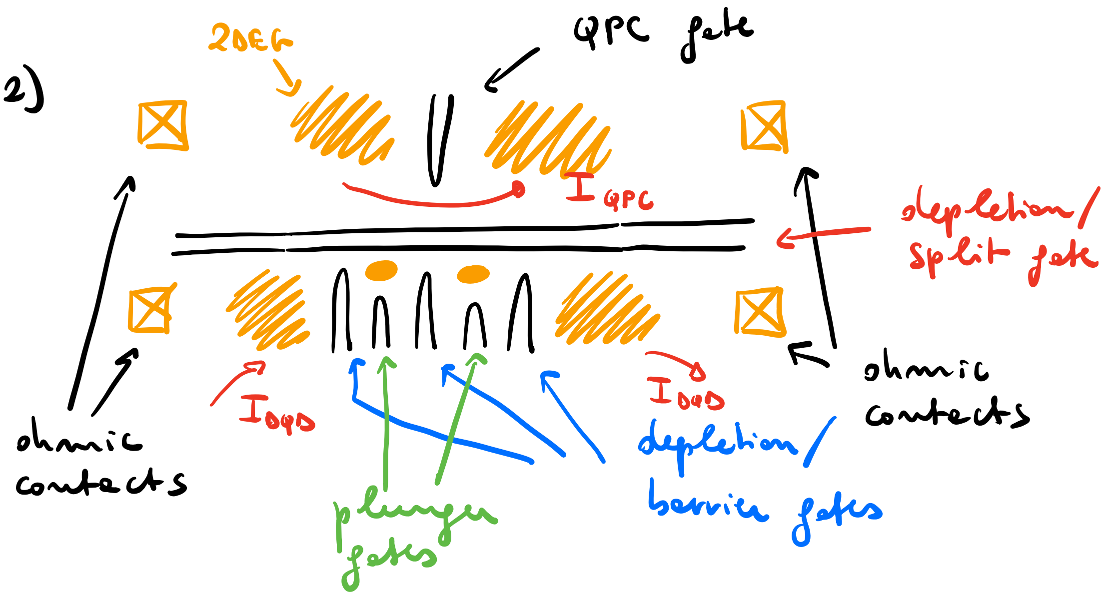
Silicon is used as a dopant for the top AlGaAs. Silicon impurities in the AlGaAs lattice introduce free electrons whose energy lies at higher energy levels than the conduction band of GaAs. As a consequence, electrons at the n^+-AlGaAs / GaAs interface, which are free to move, diffuse down to the conduction band of GaAs. Now, in the n^+-AlGaAs there is a region which is not neutral anymore because of the lack of electrons. In this region, called depletion region, only the positively charged Si^+ ions are left. According to Poisson equation, $\Delta \Phi = -\frac{\rho}{\varepsilon}$, where ρ is the charge density and Φ the potential drop, a non-zero net charge will induce a band-bending.



where the conduction band of GaAs lies below the Fermi level, a 2DEG is accumulated. This phenomenon is called DELTA DOPING or REMOTE DOPING because the dopants (the impurities) are NOT directly in the layer where the electrons are accumulated, which means that the mobility can be extremely high (> 10 millions $\frac{cm^2}{V \cdot s}$).

xy confinement → metallic gates to locally deplete the 2DEG to define QDs and/or to change the QDs chemical potential.

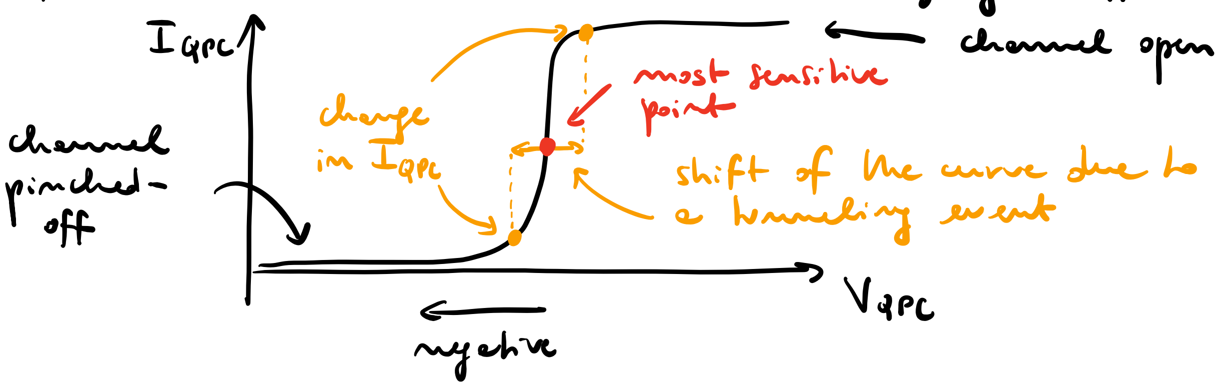
2)



- Ohmic contacts → low-resistive and linear (in bias voltage) metal-to-semiconductor contacts. They allow to ground the 2DEG (the reservoirs) so that their potential is defined and not floating and to apply bias voltage to measure a current from source to drain.
- Depletion gates → they deplete the 2DEG underneath, through negative voltages, to define the double quantum dot (DQD), in combination with the plunger gates. The central depletion gate of the

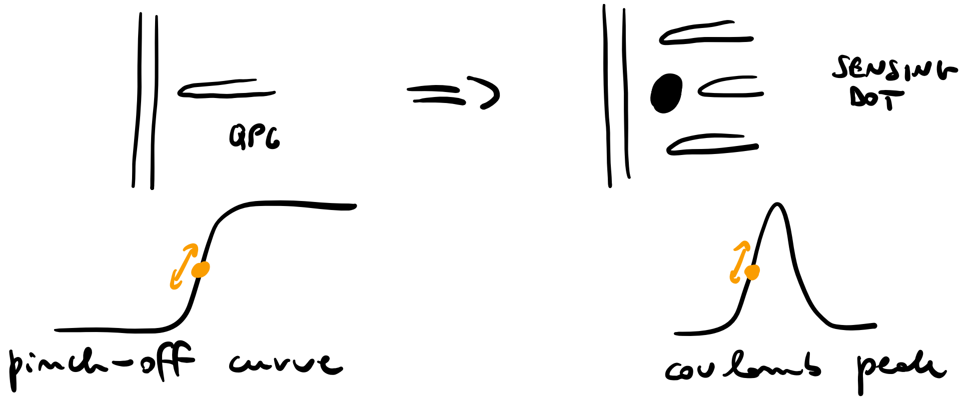
3 controls the interdot tunneling coupling, i.e. the rate at which one electron can hop from one dot to the other one and viceversa. The two side barriers control the dot-to-reservoir tunneling.

- Plunger gates → they confine the QDs in combination with the depletion gates and they control the electrons' number in the QDs.
- Depletion / split gate → it splits the QDs array from the QPC (quantum point contact) in such a way that the two currents (I_{DQD} and I_{QPC}) can be monitored independently.
- QPC gate → this gate is able to pinch-off the QPC channel by applying negative voltages. The voltage of the QPC gate is parked at the most sensitive point, so that tunneling events in the DQD can shift this curve, thus changing I_{QPC}.



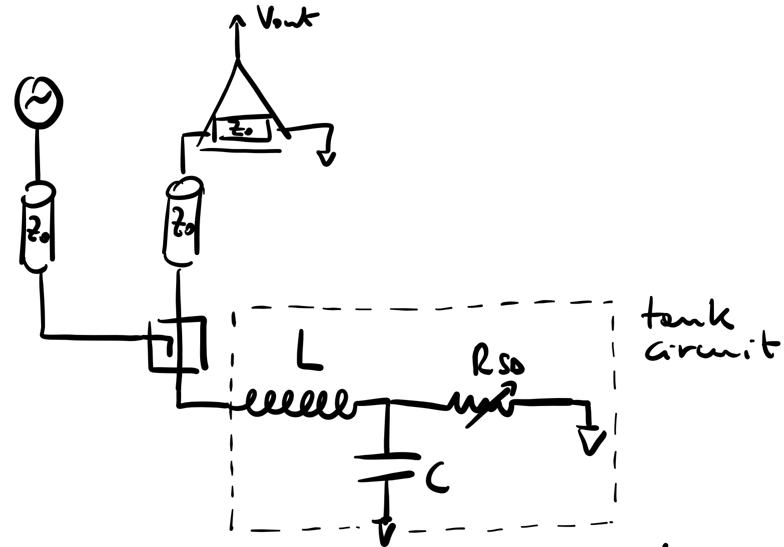
To improve the sensitivity of the QPC used for readout, the QPC can be replaced by a sensing dot.

In this way, the chemical potential of the sensing dot is much more sensitive to tunneling events in the DQD than the shift of the pinch-off curve of the QPC.



3) For spin manipulation a STRIPLINE is missing for electron spin resonance (ESR). A current flows through the stripline to generate a magnetic field to manipulate the spin state. The stripline is made of a superconducting material such that there is no thermal dissipation which can be detrimental for a system operating at 10 mK. Notice that in a system like GaAs, the spin-orbit interaction can be used to manipulate spins only via electrical signals applied through the gates already shown in the figure.

EXERCISE 4: REFLECTOMETRY



1) $$Z_{tank} = i\omega L + \left(i\omega C + \frac{1}{R_{SD}}\right)^{-1} =$$

$$= i\omega L + \left(\frac{i\omega C R_{SD} + 1}{R_{SD}}\right)^{-1} = i\omega L + \frac{R_{SD}}{1 + i\omega C R_{SD}} =$$

$$= \frac{i\omega L - \omega^2 L C R_{SD} + R_{SD}}{1 + i\omega C R_{SD}}$$

The matching condition has to happen at the resonator resonance frequency $\omega_0 = \frac{1}{\sqrt{LC}}$

$$Z_{tank}(\omega = \omega_0) = \frac{i\frac{L}{\sqrt{LC}} - \cancel{R_{SD}} + \cancel{R_{SD}}}{1 + i\sqrt{\frac{C}{L}} R_{SD}} = \frac{i\sqrt{L/C}}{1 + i\sqrt{C/L}\, R_{SD}}$$

Considering that Z_0 is a real number, we have to compute $|Z_{tank}|$:

$$|Z_{tank}| = \frac{\sqrt{L/C}}{\sqrt{1+\frac{C}{L}r_{SD}^2}} = \frac{\sqrt{L/C}}{\sqrt{\frac{L+Cr_{SD}^2}{L}}} = \sqrt{\frac{L^2}{LC+C^2r_{SD}^2}} \simeq$$

$$\simeq \sqrt{\frac{L^2}{C^2r_{SD}^2}} = \frac{L}{Cr_{SD}}$$

Matching condition: $\underline{\frac{L}{Cr_{SD}} = Z_0}$

2) $\underline{L} = Z_0 C r_{SD} = 50 \cdot 0{,}6 \cdot 10^{-12} \cdot 1 \cdot 10^{5} = 30 \cdot 10^{-7} = \underline{3\,\mu H}$